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1975 J. Phys. A: Math. Gen. 8 987

(http://iopscience.iop.org/0305-4470/8/6/018)

J. Phys. A: Math. Gen., Vol. 8, No. 6, 1975. Printed in Great Britain. © 1975

The electrostatic self-focusing process in electron streams

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Received 27 May 1974, in final form 16 December 1974

Abstract. Properties of electrostatically self-focused electron streams are calculated from stream injection parameters. The unneutralized ion density within the stream can be nearly an order of magnitude greater than that of stream particles alone. This effect is due to ionization by stream particles and collective behaviour of the stream and ambient plasma. Cold, uniform streams can occur only where stream linear density is sufficiently high. In certain cases ion cluster and molecular ion formation give rise to tight focusing. Periodic self-focusing provides axial electric fields which can resonantly drive stream particle oscillations and which can be used for acceleration of ambient ions.

1. Introduction

Electrostatic self-focusing of an electron stream results from the establishment of radial electric fields in the vicinity of the stream due to a net positive electrostatic charge accumulation in that region. Redistribution of the ionization produced by stream particle–neutral collisions gives rise to this space charge. These fields, which focus the stream, simultaneously eject positive ions from the stream region. Steady state obtains when this self-repulsion by the positive ions results in ion depletion by radial motion from the stream channel at a rate equal to the ion production rate within the channel. This phenomenon is observable in low-current (mA), low-voltage (kV) streams passing through a low-pressure (10^{-5}–10^{-3} Torr) gas.

Although the behaviour of electrostatically self-focused electron streams has been investigated for a number of years, only recently has an adequate description emerged (McCorkle and Bennett 1972). That work pointed out the strong focusing which occurs in these streams. Using an assumed radial functional dependence for the stream density, self-consistent properties were obtained from which values for the stream diameter could be calculated.

This work presents criteria for the occurrence of electrostatic self-focusing in electron streams and thereby justifies the assumption of the earlier work. The net charge density within the stream is calculated. Moreover, dynamics of non-uniform diameter streams are given. Previously unrecognized processes which produce strong focusing and which can convert energy of self-focusing into stream oscillations and axial acceleration of ambient ions are discussed.

2. Uniform diameter streams

Consider an electron stream with a current of several milliamperes and with particle energies higher than about 0·1 keV drifting through a gas of pressure in the range

10^{-5}–10^{-3} Torr approximately, producing ambient ionization by collision and experiencing radially directed forces due to the resulting space-charge electric field. In order to establish conditions for the occurrence of electrostatic self-focusing and to define stream equilibrium, a virial for the stream is useful. For a cylindrical stream the moment of inertia of a stream electron about the axis, mr^2, is related to the radial force component exerted on the particle (McCorkle and Bennett 1972) by

$$\frac{1}{4}\frac{d^2(mr^2)}{dt^2} = \frac{m}{2}\dot{r}^2 + \frac{m}{2}r^2\dot{\theta}^2 - \frac{e}{2\epsilon_0}\sum_\alpha e_\alpha \int_0^r n_\alpha(s)s\,ds. \tag{1}$$

The sum extends over all types of charged particles present in the stream (stream electrons, ambient electrons and ions), each with particle density n_α and charge e_α.

Multiplying by the distribution function for stream particles and integrating over velocity and position space per unit stream length gives

$$\frac{1}{4}\frac{d^2 I}{dt^2} = N_b\left(\frac{m}{2}\langle\dot{r}_{b0}^2\rangle + \psi_b + \theta_b\right) - \frac{\pi e}{\epsilon_0}\sum_\alpha e_\alpha \int_0^\infty \int_0^r n_\alpha(s)s\,ds\,n_b(r)r\,dr. \tag{2}$$

Here, I is the moment of inertia of all stream particles in a unit length, $(m/2)\langle\dot{r}_{b0}^2\rangle$ is the energy due to the radial component of mass motion $\dot{r}_{b0}$, averaged over the number of stream particles per unit length N_b, and ψ_b and θ_b represent energy attributable to random velocity components in the radial and azimuthal directions respectively. (The random velocities of stream particles about their average stream velocities (both radial and axial) need not be Maxwellian. In practice such 'finite-temperature streams' arise due to random behaviour in the stream emission and formation region as well as to phase mixing (Bennett 1955) and collision effects within the stream itself.)

As previously recognized (Bennett 1955), streams will concentrate towards the axis (self-focus) if $d^2I/dt^2 < 0$:

$$N_b\chi_b < \frac{\pi e}{\epsilon_0}\sum_\alpha e_\alpha \int_0^\infty \int_0^r n_\alpha(s)s\,ds\,n_b(r)r\,dr. \tag{3}$$

χ_b is the mean kinetic energy per stream particle due to velocity components transverse to the stream direction. Uniform diameter streams, for which $\dot{r}_{b0} = 0$, are characterized by the criterion

$$N_b\chi_{be} = \frac{\pi e}{\epsilon_0}\sum_\alpha e_\alpha \int_0^\infty \int_0^r n_\alpha(s)s\,ds\,n_b(r)r\,dr. \tag{4}$$

From this requirement the existence of uniform diameter streams for which the net space-charge density in the stream is proportional to the stream charge density is established. The substitution $\Sigma_\alpha e_\alpha n_\alpha(r) = fen_b(r)$ in (4) gives

$$f = 8\pi\epsilon_0\chi_{be}/e^2 N_b. \tag{5}$$

Thus uniform diameter streams are describable by

$$\sum_\alpha e_\alpha n_\alpha(r) = 8\pi\epsilon_0\chi_{be}n_b(r)/eN_b. \tag{6}$$

In the following, $\chi_{be} = 2\theta_{be} = 2\psi_{be}$ is taken to be independent of radius.

The energy spectra of the ionization collision products have a direct influence on stream focusing. Positive ions, having received negligible energy during the ionization process, emerge radially from the stream region under the influence of the electric field and experience insignificant collisional interaction during passage to the tube walls except for resonance charge exchange collisions with parent gas particles. Ambient electrons, unlike the ions, emerge from the ionization process with a considerable spread of energies. The outward flux of electrons in and around the stream consists of a drift motion superimposed on a random motion, the energy of which is spatially dependent and is related to the average energy acquired during the ionizing collision by the ejected electrons.

Using previously developed models (McCorkle and Bennett 1972) for the charged particle distributions (for which, in the interest of simplicity, only one type of ion was considered) in (6) gives

$$r = \frac{\int_0^r \mathrm{d}s(\nu_i n_b(s)s + \psi(s)) \exp(n_n B(r|s)) [(2e/M)(\phi(s)-\phi)]^{-1/2}}{n_{e0} \exp[5e\phi/(2E_0+2e\phi)] + (1+f)n_{b0} \exp(e\phi/\chi_{be})}. \tag{7}$$

Here

$$\psi(s) = n_n \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \dot{r}\sigma_{ex}(\dot{r}^2) f_i(s, \dot{r}, \dot{\theta}, \dot{z}) s^2 \,\mathrm{d}\dot{r}\,\mathrm{d}\dot{\theta}\,\mathrm{d}\dot{z} \tag{8}$$

and

$$B(r|s) = \int_r^s \sigma_{ex}(2e(\phi(s)-\phi(x))/M)\,\mathrm{d}x, \tag{9}$$

in which ν_i is the ionization collision frequency for stream electrons colliding with ambient neutrals, M is the ion particle mass, $\phi(r)$ is the electrostatic potential with $\phi(0) = 0$, σ_{ex} is the ion–neutral charge exchange cross section, n_n is the neutral particle density, f_i is the ion distribution function and E_0 is the average energy acquired by a bound electron that is ejected during an ionizing collision. E_0 may be obtained from the stream energy and ambient gas type (McCorkle and Bennett 1972). Furthermore, n_{e0} and n_{b0} are the ambient electron and stream particle densities respectively on the stream axis. For a considerable range in gas pressure, charge exchange effects on the ion distribution can be small, giving

$$r = \frac{\int_0^r \mathrm{d}s \nu_i n_b(s) s [(2e/M)(\phi(s)-\phi)]^{-1/2}}{n_{e0} \exp[5e\phi/(2E_0+2e\phi)] + (1+f)n_{b0} \exp(e\phi/\chi_{be})}. \tag{10}$$

More tractable solutions are obtained from Poisson's equation in which equation (6) is used to express the net space-charge density in terms of the stream particle density, and the stream distribution $n_b = n_{b0} \exp(e\phi/\chi_{be})$ (McCorkle and Bennett 1972) is used to express the electrostatic potential in terms of the stream particle density:

$$N_b \frac{\partial}{\partial r}\left(r \frac{\partial \ln n_b}{\partial r}\right) = -8\pi r n_b. \tag{11}$$

This result is of the form of the well known Liouville equation, a solution of which is $n_b = N_b/\pi\sigma^2(1+r^2/\sigma^2)^2$. This result in (10) gives

$$\rho = \frac{(M/\chi_{be})^{1/2} \nu_i \sigma \exp(-\eta) D(\sqrt{\eta})/2}{(n_{e0}/n_{b0}) \exp[-5\alpha\eta/(1-2\alpha\eta)] + (1+f)\exp(-2\eta)}. \tag{12}$$

Normalized variables are used: $\rho = r/\sigma$, $\eta = -e\phi/2\chi_{be}$ and $\alpha = \chi_{be}/E_0$. $D(x) = \int_0^x \exp(t^2)\,\mathrm{d}t$ is the Dawson function of x.

From the solution of the Liouville equation, $n_b = N_b/4\pi\sigma^2$ at $\rho = 1$. For this radial position, $\eta = \ln(n_{b0}/n_b)^{1/2} = \ln 2$. Evaluating (12) at $(\rho, \eta) = (1, \ln 2)$ gives the stream radius σ as

$$\sigma = \left(\frac{\chi_{be}}{M}\right)^{1/2} \frac{2K(\alpha)}{\nu_i}(1+f). \tag{13}$$

$K(\alpha)$ has been given previously (figure 1 of McCorkle and Bennett 1972). Formation of molecular ions or ion clusters in the stream can increase the ion mass and thereby result in the formation of quite small diameter streams. The on-axis charged particle densities become

$$n_{b0} = \frac{N_b M}{4\pi\chi_{be}}\left(\frac{\nu_i}{K(\alpha)(1+f)}\right)^2, \tag{14}$$

$$n_{e0} = n_{b0}(K(\alpha)-1)(1+f) \tag{15}$$

and

$$n_{i0} = n_{b0}K(\alpha)(1+f) \tag{16}$$

for the stream electrons, ambient electrons and ambient ions respectively. Spatial distributions for these quantities are given by

$$n_b = n_{b0}\exp(-2\eta), \tag{17}$$

$$n_e = n_{e0}\exp(-5\alpha\eta/(1-2\alpha\eta)] \tag{18}$$

and

$$n_i = n_{i0}\exp(-\eta)D(\sqrt{\eta})/\rho \tag{19}$$

with (12) rewritten as

$$\rho = K(\alpha)\exp(-\eta)D(\sqrt{\eta})\{(K(\alpha)-1)\exp[-5\alpha\eta/(1-2\alpha\eta)] + \exp(-2\eta)\}^{-1}. \tag{20}$$

The one-parameter solutions given by (20) are shown in figure 1.

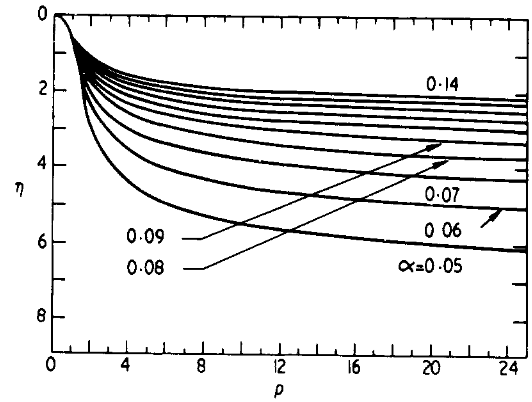

Figure 1. Universal potential profiles in and around uniform diameter electrostatically self-focusing electron streams. (The stream model $n_b = N_b/\pi\sigma^2(1+\rho^2)^2$ is inadequate for $\alpha < 0{\cdot}05$ and other solutions of the Liouville equation should be used.) Electrostatic potential is normalized to the transverse thermal spread of the stream and radial position is normalized to the stream radius.

3. Non-uniform diameter streams

The development of non-uniform diameter streams may be described by moment equations. For axially symmetric streams with particle random velocity components evenly distributed about the velocity of mass motion, $(\dot{r}_{b0}, \dot{z}_{b0})$, isotropic in components transverse to the stream axis, and subject to a force (F_r, F_z), relevant moment equations are:

$$\frac{\mathrm{d}\ln n_b}{\mathrm{d}t}+\frac{1}{r}\frac{\partial}{\partial r}(r\dot{r}_{b0})+\frac{\partial \dot{z}_{b0}}{\partial z} = 0 \tag{21}$$

$$\frac{\mathrm{d}\dot{r}_{b0}}{\mathrm{d}t}-\frac{F_r}{m}+\frac{1}{n_b}\frac{\partial}{\partial r}(n_b 2\psi_t/m) = 0 \tag{22}$$

$$\frac{\mathrm{d}\dot{z}_{b0}}{\mathrm{d}t}-\frac{F_z}{m}+\frac{1}{n_b}\frac{\partial}{\partial z}(n_b 2\psi_l/m) = 0 \tag{23}$$

$$\frac{\mathrm{d}\ln \psi_t}{\mathrm{d}t}+2\frac{\partial \dot{r}_{b0}}{\partial r} = 0 \tag{24}$$

$$\frac{\mathrm{d}\ln \psi_t}{\mathrm{d}t}+2\frac{\dot{r}_{b0}}{r} = 0 \tag{25}$$

and

$$\frac{\mathrm{d}\ln \psi_l}{\mathrm{d}t}+2\frac{\partial \dot{z}_{b0}}{\partial z} = 0 \tag{26}$$

where $\mathrm{d}/\mathrm{d}t = \partial/\partial t+\dot{r}_{b0}\partial/\partial r+\dot{z}_{b0}\partial/\partial z$. These are conservation equations for particle number, radially directed momentum, axially directed momentum and energy due to components of velocity in the radial, azimuthal and axial directions respectively. ψ_t is the energy due to random velocity components for a single degree of freedom transverse to the stream direction and ψ_l is the equivalent quantity for components along the stream.

Two results follow immediately. Adding (24), (25) and (26) and subtracting the result from (21) gives $(\mathrm{d}/\mathrm{d}t)(\ln \psi_t\psi_l^{1/2}/n_b) = 0$ or

$$\psi_t\psi_l^{1/2}/n_b = \text{constant} \tag{27}$$

along a streamline, whereas (24) and (25) give

$$\dot{r}_{b0} = -\frac{1}{2}\frac{\mathrm{d}\ln \psi_t}{\mathrm{d}t}r. \tag{28}$$

Here, $\mathrm{d}(\ln \psi_t)/\mathrm{d}t$ is a function of axial position z alone. Since $\dot{r}_{b0} \sim r$ and $\mathrm{d}r/\mathrm{d}z = \dot{r}_{b0}/\dot{z}_{b0}$ for a streamline, solutions with $\dot{z}_{b0} = \dot{z}_{b0}(z)$ preserve the stream density profile. Otherwise significant changes in stream density profiles can occur due to radial mixing (one such case is the occurrence of hollow streams for which the density is not maximum on axis).

Although many solutions of the moment equations are possible (consistent with many stream developments that occur in practice), only one characteristic set is presented here. For a stream initially Gaussian in radial disposition and with a longitudinal thermal spread near the equilibrium value (see equation (36)), ψ_l may be assumed to undergo no significant change; $\psi_l = \chi_{be}/2$. Then from (26), $\dot{z}_{b0}$ is constant and the Gaussian form is preserved throughout the motion:

$$n_b(r, z) = N_b \exp(-r^2/\sigma^2(z))/\pi\sigma^2(z). \tag{29}$$

Then (27) gives

$$\psi_t(z) = \psi_{t0}\sigma_0^2/\sigma^2(z) \tag{30}$$

for a stream injected with an initial thermal spread in the transverse direction of ψ_{t0} and with a radius σ_0 at that point. For a drifting, electrostatically self-focusing stream, $F_r = e\partial\phi/\partial r$ and $F_z = e\partial\phi/\partial z$. With a radially uniform transverse thermal spread, (22) and (23) give

$$\frac{d^2R}{ds^2} + \frac{1}{2R} - \frac{1}{R^3} = 0 \tag{31}$$

and

$$e\phi(r, z) = \chi_{be}[\ln(\sigma_0/\sigma)^2 - (r/\sigma)^2] \tag{32}$$

for the steady-state development. The normalization $R = (\sigma/\sigma_0)(\chi_{be}/\psi_{t0})^{1/2}$ and $s = (2z/\sigma_0)(\chi_{be}/\dot{z}_{b0}\sqrt{(m\psi_{t0})})$ was invoked in writing (31). This equation is of the form of the Vladimirskij–Kapchinskij equation. The solution is given in figure 2 (from the work of Garren (1969) which, although concerned with a different physical situation, gave rise to the same equation).

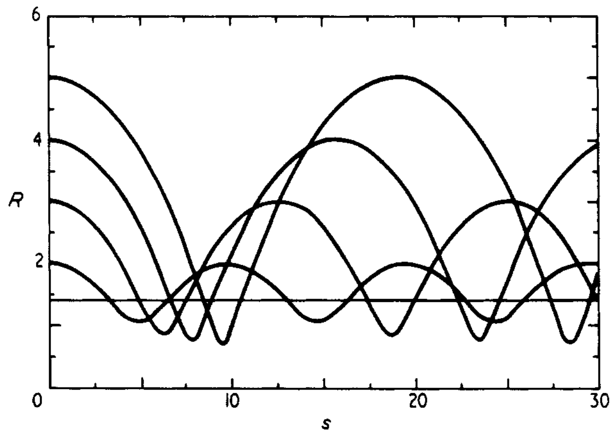

Figure 2. Axial development of stream radius for a drifting electrostatically self-focusing electron stream (solution of (31) from the work of Garren (1969)).

The first integral of (31) gives

$$\frac{1}{2}\left(\frac{dR}{ds}\right)^2 + V(R) = 0 \tag{33}$$

with

$$V(R) = \frac{1}{2}\left[\ln\left(\frac{R}{R_0}\right) + R^{-2} - R_0^{-2} - \left(\frac{dR}{ds}\right)_0^2\right]. \tag{34}$$

a form equivalent to that for the motion of a unit mass particle subject to a force field characterized by the potential $V(R)$. Here, $(\mathrm{d}R/\mathrm{d}s)_0$ is the initial angular convergence or divergence of the stream. Equilibrium occurs for $\mathrm{d}V/\mathrm{d}R = 0$ giving $R_e = \sqrt{2}$ or

$$\sigma_e = \sigma_0(2\psi_{t0}/\chi_{be})^{1/2}. \tag{35}$$

This result may be used with that given in (13) to obtain

$$\chi_{be}K(\chi_{be}/E_0)(1+8\pi\epsilon_0\chi_{be}/e^2N_b) = \sigma_0 v_i(M\psi_{t0}/2)^{1/2}, \tag{36}$$

which specifies χ_{be} in terms of the initial parameters of the stream. Moreover, since $\sigma_e \lesseqgtr \sigma_0$ if $(\mathrm{d}V/\mathrm{d}R)_{R_0} \gtreqless 0$ respectively, streams will be focusing if $\psi_{t0} < \chi_{be}/2$ and anti-focusing if $\psi_{t0} > \chi_{be}/2$. With the occurrence of radial mixing, streams evolve toward a uniform diameter. Then ψ_{t0} must be replaced by $\psi'_{t0} = \psi_{t0} + \frac{1}{2}m\dot{z}_{b0}^2(\mathrm{d}\sigma/\mathrm{d}z)_0^2$ in these equilibrium and focusing estimates. Consequently the condition for stream contraction by electrostatic self-focusing is

$$\sigma_0 v_i\sqrt{M} > 2\sqrt{(2\psi'_{t0})}K(2\psi'_{t0}/E_0)(1+16\pi\epsilon_0\psi'_{t0}/e^2N_b). \tag{37}$$

Maximum and minimum stream radii, R_M and R_m respectively, are given by $\mathrm{d}R/\mathrm{d}s = 0$ and thus by $V(R_{M,m}) = 0$:

$$R_{M,m}^{-2} + \ln R_{M,m} = R_0^{-2} + \ln R_0 + (\mathrm{d}R/\mathrm{d}s)_0^2. \tag{38}$$

The bounce length of the stream, defined as the axial distance between consecutive maxima (minima) of the stream radius, may be estimated for cases with small radial deviation from equilibrium by expanding the force term, $(2R)^{-1} - R^{-3}$, about $\sqrt{2}$ and retaining the linear term only. This gives a bounce length $\lambda_s = 2\pi\sqrt{2}$ (see figure 2) or

$$\lambda_z = \left(\frac{m}{M}\right)^{1/2}\frac{2\pi K(\alpha)}{P_i p}(1+f). \tag{39}$$

Here P_i is the probability of ionization and p the gas pressure. Finally, the radial and axial electric field values may be obtained as

$$E_r = 2\chi_{be}r/e\sigma^2 \tag{40}$$

and

$$E_z = 2\chi_{be}\frac{\mathrm{d}\sigma}{\mathrm{d}z}(1-r^2/\sigma^2)/e\sigma, \tag{41}$$

for which (33) and (34) give $\mathrm{d}\sigma/\mathrm{d}z$. Typically $E_z \sim 8\chi_{be}/e\lambda_z$ in the stream.

Although a particular stream model was utilised here, experiment shows these results hold for cases in which strong changes occur in the stream distribution (unpublished results by the author).

4. Discussion

Electrostatically self-focusing electron streams are characterized by the description given here over a broad range of parameters. The limitation incurred by ignoring scattering of stream particles may be estimated by recognizing that for stream particle energies of interest, a high percentage of scattering collisions with neutrals result in ionization. The requirement for axial development to occur as presented over axial distances of several λ_z becomes $P_i p\lambda_z \ll 1$, or using (39), (36) and (5):

$$\frac{\sigma_0 v_i \psi_{t0}^{1/2}}{f N_b} \ll \frac{e^2}{8\pi^2 \epsilon_0 \sqrt{(2m)}} \simeq 2{\cdot}72 \times 10^{-14}\,(\text{mks}). \tag{42}$$

The similar requirement for neglecting angular scattering due to Coulombic interactions, $(\mathrm{d}\epsilon_t/\mathrm{d}x)_c \lambda_z \ll \chi_{be}$ (where $(\mathrm{d}\epsilon_t/\mathrm{d}x)_c$ is the increase in transverse energy of stream particles per unit axial distance due to Coulombic scattering with ambient ions (Thomas 1928)), becomes

$$\frac{f^2 N_b}{P_i p}\left(\frac{m}{M}\right)^{1/2} \gg 4\left(\ln\left(\frac{\delta_b}{\ln \delta_b}\right) - \gamma - 1\right) \tag{43}$$

with

$$\delta_b = \frac{3\sigma_0}{N_b v_i}\left(\frac{\psi_{t0}}{2M}\right)^{1/2}\left(\frac{4\pi\epsilon_0 m \dot{z}_{b0}^2}{e^2}\right)^3. \tag{44}$$

The right-hand side of (43) approaches a value of 200 and is slowly varying. Euler's constant, γ, is 0·5772.

The ambient electron model involved neglect of collisional energy loss due to elastic and inelastic collisions with neutrals and energy exchange with ions, in comparison with energy production by stream particles in ionizing collisions (McCorkle and Bennett 1972). Power density values for these processes justify this assumption for pressures up to the order of 10^{-3}–10^{-2} Torr in gases for which metastable states do not play a dominant role.

Neglect of charge exchange in the ion density model requires that the radial component of electrostatic force experienced by ions in the streams overcomes the collisional drag force: $eE > M v_i^2 n_n \sigma_{ex}$ for an ion radial drift velocity v_i. This runaway condition is satisfied for neutral densities below $4/\sigma_e \sigma_{ex}$, the charge exchange cross section being evaluated at an energy of $\chi_{be}/2$ approximately. Similarly, for ion runaway in the presence of dynamical friction due to ion–ion interactions, there obtains

$$\frac{f^5 N_b^3}{M v_i^2} > \frac{10^4 \pi \epsilon_0}{e^2}\left(\ln\left(\frac{\delta_i}{\ln \delta_i}\right) - \gamma - 1\right). \tag{45}$$

Here, $\delta_i = (kT_n/m\dot{z}_{b0}^2)^3 \delta_b$ with an ambient gas temperature T_n.

Within the model employed there is a fundamental limitation on the strength of electrostatic self-focusing as well as a range of parameters for which stream and ambient particles can act collectively to produce this focusing. In the first case, stream energies in transverse velocity components are limited by the requirement $\alpha < 0{\cdot}1433$ (in order that $K(\alpha) < \infty$). Consequently,

$$\chi_{be} < 0{\cdot}1433\, E_0. \tag{46}$$

For stream particles to exhibit collective behaviour, the stream radius should be comparable to or exceed the Debye length for these particles. Then

$$f \leqslant 8 \qquad \text{or} \qquad N_b \geqslant \pi\epsilon_0 \chi_{be}/e^2. \tag{47}$$

This restriction removes the anomalous result from (13) that as the stream current becomes small ($N_b \to 0$), the stream radius diverges ($f \to \infty$). In fact, for stream particle line densities below the limit given in (47), the stream begins to behave as a collection of particles. The similar requirement that ambient electrons behave collectively gives

$$K(\alpha) \geqslant 1+\frac{1}{20\alpha}\left(\frac{f}{1+f}\right). \tag{48}$$

Cooperative ion behaviour is ensured in this limit too. A display of these restrictions is given in figure 3.

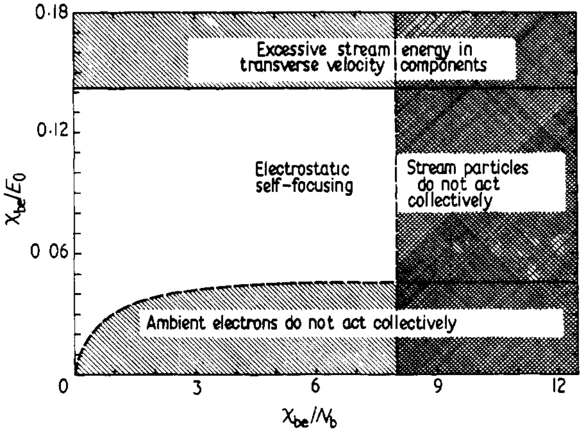

Figure 3. Region of operation for well-behaved electrostatically self-focusing electron streams. (The unit of χ_{be}/N_b is $e^2/8\pi\epsilon_0 = 1{\cdot}15\times10^{-28}$ Jm.)

Potential values assumed by a cylindrical tube coaxially surrounding such streams, and the consequent wall–sheath formation, have been characterized previously (McCorkle and Bennett 1972).

Conversion of electrostatic self-focusing energy into stream particle oscillations may occur. The longitudinal component of the electrostatic field of a bouncing stream presents a time-varying field of frequency $2\pi\dot{z}_{b0}/\lambda_z$ in the frame in which stream particles are at rest. Thus a resonant condition obtains when this value equals the plasma frequency of the stream particles, $N_b e^2 = 4\pi\epsilon_0\chi_{be}$ or $f = 2$. Moreover, the longitudinal field component may provide significant axial acceleration of ambient ions. Since the axial field obtains values approaching $4\pi\chi_{be}/e\lambda_z$ at certain axial positions, the longitudinal force on a singly charged ion is

$$F_z \simeq (e^2 f N_b/4\pi\epsilon_0)^2/\sigma_0\dot{z}_{b0}(2m\psi_{t0})^{1/2}. \tag{49}$$

Runaway ion behaviour then obtains provided that

$$\frac{f^2 N_b k T_n}{\dot{z}_{b0} v_i\sqrt{M}} > 2\sqrt{(m)}\left(\ln\left(\frac{\delta_i}{\ln\delta_i}\right)-\gamma-1\right) \tag{50}$$

and

$$p < F_z/3\sigma_0\sigma_{ex}, \tag{51}$$

the charge exchange cross section being evaluated at an energy kT_n. Obviously the axial position of the force maximum may be made time-dependent by varying the initial angular convergence (or divergence) of the stream. Some self-synchronization of the acceleration process may also occur (Putnam 1970).

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